Multistage Model for Distribution Expansion Planning with Distributed Generation—Part II: Numerical Results

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***Abstract*—This paper presents the computer simulation of the Multistage Model for Distribution Expansion Planning with Distributed Generation, as described in Part I. The simulations deal with the planning of an electrical power distribution network in three stages, in five different situations: 1) each of the three stages planned independently; 2) multistage planning; 3) multistage planning with distributed generation; 4) multistage planning with distributed generation and constraints on investment; and 5) multistage planning with distributed generation considering three load levels. The influence of additional constraints is analyzed in terms of the computational effort required to find the optimum solution to the problem.**

***Index Terms*—Distributed generation (DG), power distribution, power distribution economics, power distribution planning.**

I. Introduction

THE problem of how to plan the expansion of a distribution system has been the subject of much recent research. Different approaches are presented in the literature, varying from model structures to the methods used for problem solutions [2]–[5]. A short description of the models and methods used to solve the problem has been given in [1]. The model used in this paper aims at finding the best solution to the problem of planning in multiple stages, taking into account the influence of distributed generation (DG). The problem is formulated in terms of mixed integer programming and solved by using mathematical methods of optimization. However, the combinatorial characteristics of the problem are such that it is difficult to use mathematical optimization methods directly, the model proposed here includes constraints which bring to the general problem some of the characteristics encountered in the practical operation of distribution systems. These constraints limit the search space and considerably reduce the computational effort required to find the solution to the problem. Logical constraints are imposed that express the practical limitations on network investment and operation, together with two kinds of additional constraints related to the network topology, namely: 1) constraints on new paths and 2) fence constraints. Results from the examples given in this paper were obtained using branch-and-bound algorithms. The solvers used are available in the network-enabled optimization system (NEOS) [6].

This paper is organized as follows. Section II describes the distribution network used to test the proposed model. Section III gives results obtained using stage-by-stage sequential planning, multistage planning, and multistage planning with the existence of DG included. Section IV evaluates the reduction in computer effort used to solve the problem when additional constraints on new paths and fencing constraints are imposed. This paper ends with a summary of conclusions.

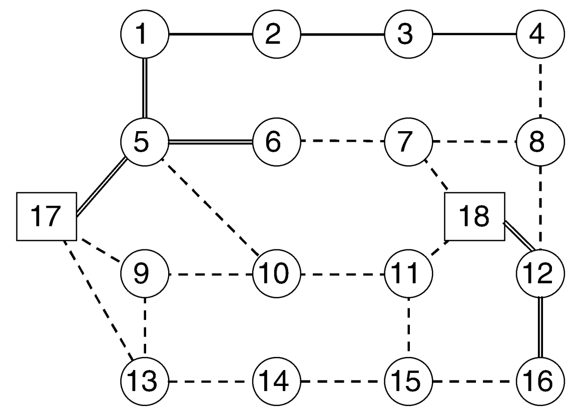

Fig. 1. Diagram of the 18-node network.

II. Distribution Network

To validate the mathematical model given in the first part of this paper [1], a fictitious three-phase network was used consisting of 18 nodes (2 substations and 16 nodes with loads) and 24 branches operating under 13800 V. The topology of this network is shown in Fig. 1 in which rectangles denote the substations, circles are the nodes where loads are concentrated, branches drawn as continuous lines denote the initial network (those with a single line are part of the fixed network and those with double lines are candidates for replacement), and branches drawn as dashed lines are candidates for addition (and are not part of the initial network). The basis values for the whole network are 1 MVA and 13.8 kV.

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TABLE II
BRANCH DATA OF THE 18-NODE NETWORK

Node		Initial		Option 1			Option 2			Option 3		
from	to	$f_{\max}$ [A]	Z [Ω]	$f_{\max}^1$ [A]	Z^1 [Ω]	C^1 [\$]	$f_{\max}^2$ [A]	Z^2 [Ω]	C^2 [\$]	$f_{\max}^3$ [A]	Z^3 [Ω]	C^3 [\$]
Installed network												
1	2	250	1.0	—	—	—	—	—	—	—	—	—
2	3	250	1.0	—	—	—	—	—	—	—	—	—
3	4	250	1.0	—	—	—	—	—	—	—	—	—
Replacement network												
1	5	250	1.0	400	0.7	20	500	0.5	38	—	—	—
5	6	250	1.0	400	0.7	21	500	0.5	39	—	—	—
5	17	250	1.0	400	0.7	18	500	0.5	36	—	—	—
12	16	250	1.0	400	0.7	22	500	0.5	40	—	—	—
12	18	250	1.0	400	0.7	19	500	0.5	37	—	—	—
Addition network												
4	8	—	—	250	1.0	90	400	0.7	110	500	0.5	130
5	10	—	—	250	1.0	92	400	0.7	112	500	0.5	132
6	7	—	—	250	1.0	94	400	0.7	114	500	0.5	134
7	8	—	—	250	1.0	96	400	0.7	116	500	0.5	136
7	18	—	—	250	1.0	300	400	0.7	320	500	0.5	350
8	12	—	—	250	1.0	98	400	0.7	118	500	0.5	138
9	10	—	—	250	1.0	100	400	0.7	120	500	0.5	140
9	13	—	—	250	1.0	102	400	0.7	122	500	0.5	142
9	17	—	—	250	1.0	305	400	0.7	325	500	0.5	355
10	11	—	—	250	1.0	104	400	0.7	124	500	0.5	144
11	15	—	—	250	1.0	106	400	0.7	126	500	0.5	146
11	18	—	—	250	1.0	310	400	0.7	330	500	0.5	360
13	14	—	—	250	1.0	108	400	0.7	128	500	0.5	148
13	17	—	—	250	1.0	315	400	0.7	335	500	0.5	365
14	15	—	—	250	1.0	110	400	0.7	130	500	0.5	150
15	16	—	—	250	1.0	112	400	0.7	132	500	0.5	152

TABLE I
NODE DATA FOR THE 18-NODE NETWORK

Node #	d [MVA]								
	Stage 1			Stage 2			Stage 3		
	LL1	LL2	LL3	LL1	LL2	LL3	LL1	LL2	LL3
1	**1.2**	0.72	0.24	**1.2**	0.72	0.24	**1.2**	0.72	0.24
2	—	—	—	**1.2**	0.72	0.24	**1.2**	0.72	0.24
3	—	—	—	**1.2**	0.72	0.24	**1.2**	0.72	0.24
4	**1.2**	0.72	0.24	**1.2**	0.72	0.24	**1.2**	0.72	0.24
5	**1.2**	0.72	0.24	**1.2**	0.72	0.24	**1.2**	0.72	0.24
6	**1.2**	0.72	0.24	**1.2**	0.72	0.24	**1.2**	0.72	0.24
7	—	—	—	**1.2**	0.72	0.24	**1.2**	0.72	0.24
8	**1.2**	0.72	0.24	**1.2**	0.72	0.24	**1.2**	0.72	0.24
9	**1.2**	0.72	0.24	**1.2**	0.72	0.24	**2.4**	1.2	0.48
10	—	—	—	1.2	**2.4**	0.48	2.4	**3.6**	1.2
11	**1.2**	0.72	0.24	1.2	**2.4**	0.48	2.4	**3.6**	1.2
12	**1.2**	0.72	0.24	**1.2**	0.72	0.24	**1.2**	0.72	0.24
13	**1.2**	0.72	0.24	1.2	**2.4**	0.48	2.4	**3.6**	1.2
14	—	—	—	**1.2**	0.72	0.24	**2.4**	1.2	0.48
15	—	—	—	**1.2**	0.72	0.24	**2.4**	1.2	0.48
16	**1.2**	0.72	0.24	**1.2**	0.72	0.24	**1.2**	0.72	0.24
Hours per day	3	13	8	3	13	8	3	13	8

Table I gives the node loads for the three stages leading to the planning horizon. Each stage considers three load levels, describing a typical daily load curve. The load level 1 (LL1) represents the part of the typical day with maximum power consumption (peak-hour). The load level 2 (LL2) represents the power consumption in the major part of the day. The load level 3 (LL3) represents the period with lower power consumption. The maximum power injection of the existing substation at nodes 17 and 18 are 12 MVA for stage 1, and 24 MVA for stages 2 and 3.

The data of the branches are given in Table II, where the columns show the capacities and impedances of branches in the initial network, and the capacities, impedances, and costs of the branches that are candidates for replacement or addition. The annual cost of operation and maintenance (O_i^{FI}, O_j^{RJ}, and O_k^{AK}) was assumed as 1 for all branches; the cost of energy not supplied (C_m^D) was set at $4.2 \times 10^6\ ^{\$}/_{\text{MVA}}$. This high cost for energy not supplied is used to avoid load shedding. The planning horizon is four years, divided into three stages, the first two being one-year duration and the third being for two years. The first stage starts at the base year. The annual rate of interest on capital was set at 10%, with present value factors for the costs of investment and operation given by $\delta_1^{\text{inv}} = \delta_1^{\text{oper}} = 1, \delta_2^{\text{inv}} = \delta_2^{\text{oper}} = 0.9091, \delta_3^{\text{inv}} = 0.8264e\delta_3^{\text{oper}} = 1.5778$ [1, (1.3) and (1.4)]. The voltage limits are $V_{\min} = 13110$ V and $V_{\max} = 14490$ V.

The optimization problem has 58 binary variables for investment (two cable options for the five-branches candidates for replacement and three cable options for the 16 branches candidates for addition to the network) and 66 utilization variables (one option for the three network branches already in existence; the initial cable plus two options for the 16 branches that are candidates for replacement; and three cable options for the 16 branches that are candidates for addition), yielding 124 binary variables for each stage.

In what follows, five cases are analyzed in order to evaluate the model:

Case 1) stage-by-stage planning without distributed generation ($g_{\max,10,t}^G = 0$);

Case 2) multistage planning without distributed generation;

Case 3) multistage planning with distributed generation;
Case 4) multistage planning with distributed generation and constraints on investment;
Case 5) multistage planning with distributed generation considering three load levels.

For the four initial cases, 1) to 4), the peak nodal load values, written in boldface in Table I, are considered for the planning. The results presented were obtained by using the solver Xpress-MP with the NEOS [6], with default parameter settings.

III. Numerical Results

A. Year-by-Year Planning

For this case, stages were planned one after the other, taking the solution obtained for expansion in the preceding stage as the starting point. Fig. 2 shows the investments selected (indicated at the respective nodes by the letters $A1$ for addition by option 1, $R2$ for replacement by option 2), the nodal voltages at the ends of the branches, the power injection at the substations, and the costs of each stage, yielding 1488.79 for the present value of the total cost. To reduce the expansion cost of each stage taken separately, some investments are made in stages 1 and 2 which become obsolete in the succeeding stages: namely, the addition of branches 9–10, 10–11, and 14–15, and replacement of the branch 5–17 by option 1. The effect of the voltage limits is shown by the replacement option $R2$ in branch 12–16. Although at stage 3 the current through branch 12–16 is 300 A, the option $R1$ was not used (cheaper, but with greater impedance), with a capacity for 400 A, in order that the voltage at node 11 should not violate its lower limit.

B. Multistage Planning

For this case, planning was considered by taking all stages together. The solution obtained and the costs of each stage are shown in Fig. 3, yielding 1162.48 for the present value of the total cost. Although the investment at stage 1 is greater than that obtained under the year-by-year expansion of Fig. 2, the total cost is about 22% less. The decision to install a new feeder (branch 9–17) is anticipated during the first stage and no investments are made that become obsolete later, as the planning takes the longer term approach.

C. Multistage Planning With DG

For this case, planning allowed the possibility of DG with generation at node 10, at a cost of $4.2^{\$}/_{\mathrm{MVA}}$ and available capacity of 1.2 MVA, 2.4 MVA, and 7.2 MVA, respectively, in stages 1, 2, and 3. The solution obtained and the costs of each stage are shown in Fig. 4, where the present value amounts to 1040.82. Although DG is not used during the first stage, in the following stages, the use of generation at node 10 avoided the need to install a new feeder, as occurred in former cases. The load at nodes 9, 13, and 14 is shared between existing feeders and the total cost is reduced by 10% relative to that given in Fig. 3. It can be seen that in stage 3, the DG at node 10, calculated as 6 MVA, was determined so that the voltage at node 13 should not violate the lower limit set at 13110 V. It can also be seen that the path defined by nodes 18-12-16-15 was dimensioned using the option with cables of lower impedance (the most expensive option), so that the voltage at node 11 satisfied its lower limit in stage 3.

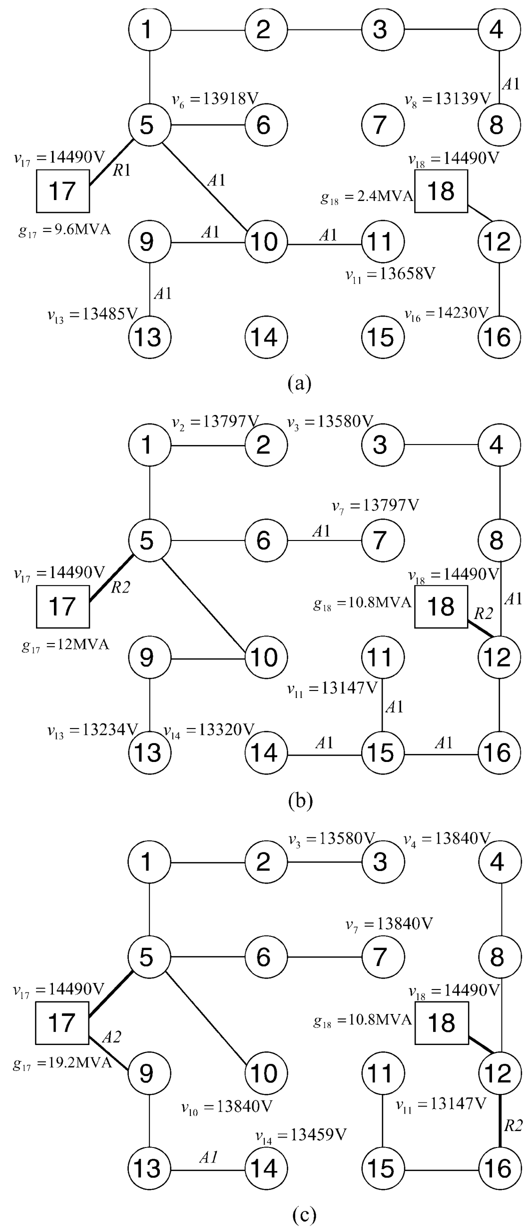

Fig. 2. Solution with year-by-year expansion: $C = 1488.79$. (a) Stage 1: $c_1^{\mathrm{inv}} = 506.00$ and $c_1^{\mathrm{oper}} = 13.00$. (b) Stage 2: $c_2^{\mathrm{inv}} = 593.00$ and $c_2^{\mathrm{oper}} = 16.00$. (c) Stage 3: $c_3^{\mathrm{inv}} = 473.00$ and $c_3^{\mathrm{oper}} = 16.00$.

D. Multistage Planning With DG and Constraint on Investment

For this case, the planning considered that DG capacity existed at node 10, at a cost of $4.2^{\$}/_{\mathrm{MVA}}$ and available capacity of 1.2, 2.4, and 7.2 MVA, respectively, in stages 1, 2, and 3. The investment available at each stage is restricted to 600, so that the investment proposed at stage 1 in the solution given in Fig. 4 is no longer feasible. The solution obtained and the costs of each stage are shown in Fig. 5, where the present value amounts to 1075.91. It can be seen that the DG at node 10 was used only in stage 3, with a maximum value of 7.2 MVA. In contrast with the solution found for the preceding case (Fig. 4), node 11 was supplied by the substation at node 17, as the investment needed

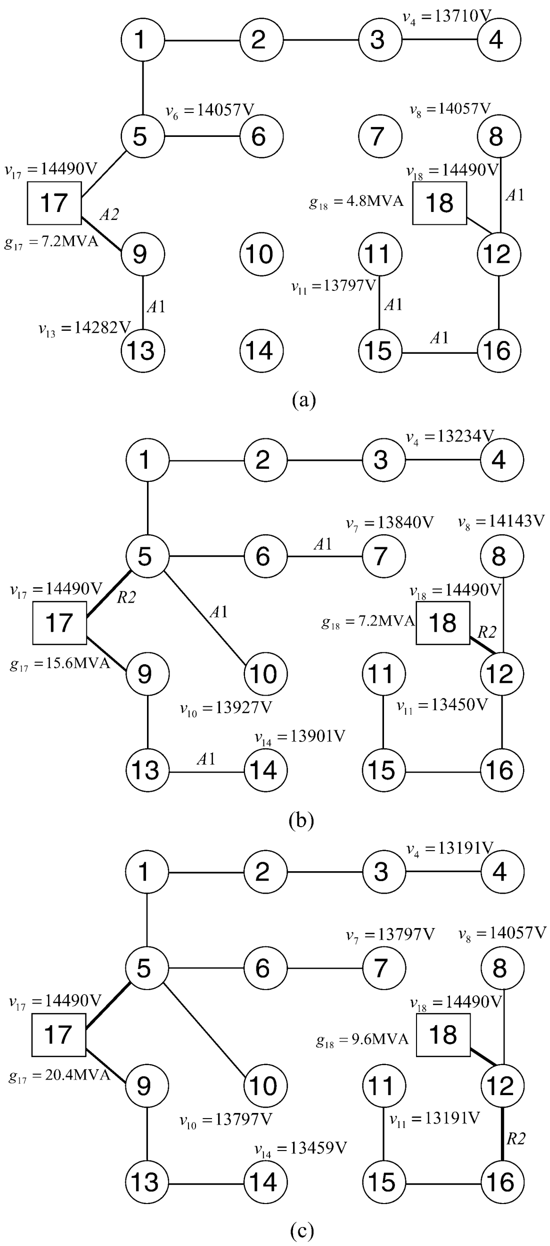

Fig. 3. Solution with multistage expansion: $C = 1162.48$. (a) Stage 1: $c_1^{\mathrm{inv}} = 743.00$ e $c_1^{\mathrm{oper}} = 13.00$. (b) Stage 2: $c_2^{\mathrm{inv}} = 367.00$ e $c_2^{\mathrm{oper}} = 16.00$. (c) Stage 3: $c_3^{\mathrm{inv}} = 40.00$ e $c_3^{\mathrm{oper}} = 16.00$.

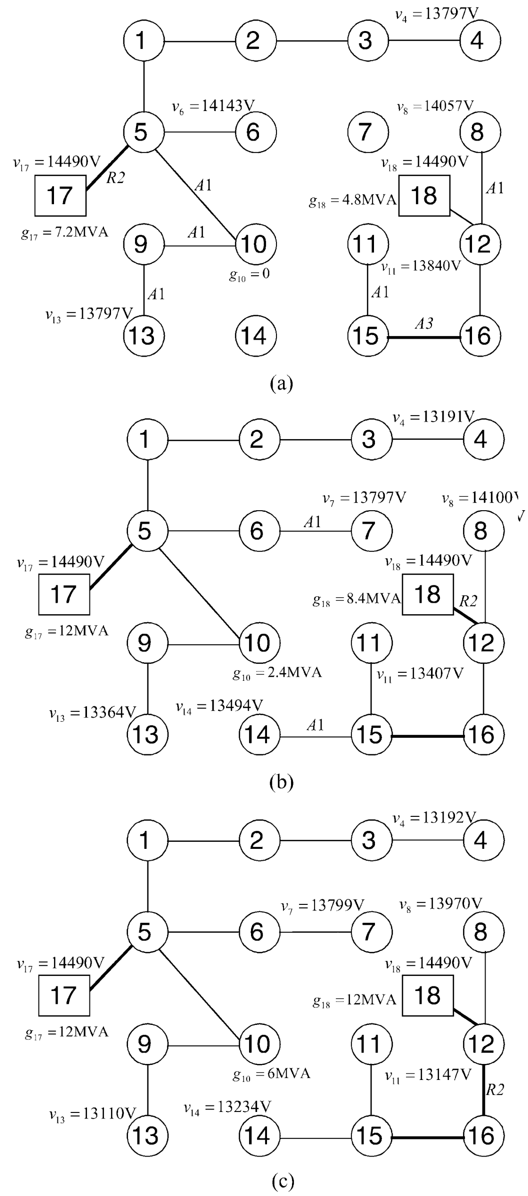

Fig. 4. Solution for multistage expansion with distributed generation: $C = 1040.82$. (a) Stage 1: $c_1^{\mathrm{inv}} = 686.00$ e $c_1^{\mathrm{oper}} = 14.00$. (b) Stage 2: $c_2^{\mathrm{inv}} = 241.00$ e $c_2^{\mathrm{oper}} = 26.00$. (c) Stage 3: $c_3^{\mathrm{inv}} = 40.00$ e $c_3^{\mathrm{oper}} = 41.20$.

for constructing the path consisting of nodes 16-15-11 (258.00) is significantly greater than the option given by the path through nodes 10-11 (104.00). In this case, the solution found uses the fact that node 10 is already used to satisfy the loads at nodes 9 and 13.

E. Multistage Planning With DG Considering Three Load Levels

For this case, each stage of the planning horizon is replaced by three simultaneous stages, in order to represent the load levels shown in Table I. Similar to the cases described in Section III-C, the planning includes DG capacity at node 10, at a cost of $21^{\$}/_{\mathrm{MVA}}$ for the LL1, due the peak-hour, and $4.2^{\$}/_{\mathrm{MVA}}$ for LL2 and LL3. The solution obtained and the costs of each stage are shown in Fig. 6 and Table III, where the present value of the cost amounts to 940.76. In this case, a solution with lower cost was obtained, when compared to the solution shown in Fig. 4, in which only the peak load has been considered for each stage. In a real situation, the maximum of nodal loads does not occur at the same time. Therefore, there is no reason to plan the network to satisfy these unrealistic load conditions. As shown in Table III, the DG capacity was used on the second and third stages which represent the heaviest load conditions—LL1 and LL2.

IV. Analysis of the Results

The multistage problem given in Section III-B has 372 binary variables (124 binary variables at each stage), resulting in 2^{372} combinations. When the set of logical constraints is introduced [1, (18)–(27)], the search space is dramatically reduced, falling to approximately 2^{118} combinations. The additional new-path constraints and fencing constraints, also given in [1], depend

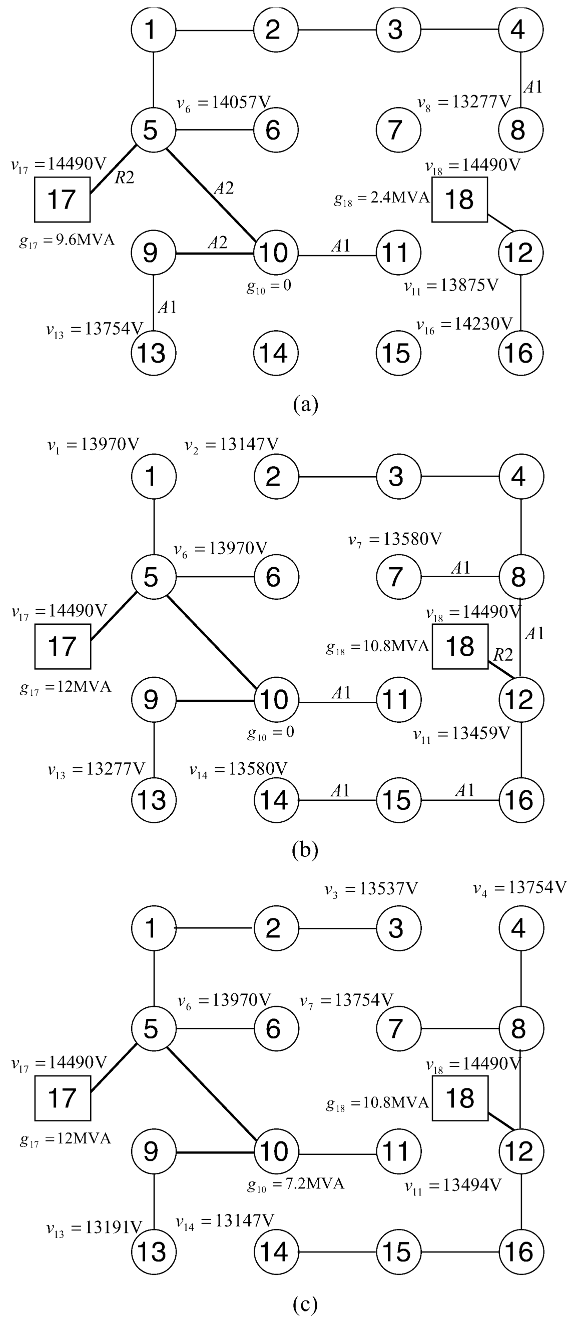

Fig. 5. Solution for multistage expansion with distributed generation and constrained investment: $C = 1075.91$. (a) Stage 1: $c_1^{\mathrm{inv}} = 564.00$ and $c_1^{\mathrm{oper}} = 13.00$. (b) Stage 2: $c_2^{\mathrm{inv}} = 453.00$ and $c_2^{\mathrm{oper}} = 16.00$. (c) Stage 3: $c_3^{\mathrm{inv}} = 0.00$ and $c_3^{\mathrm{oper}} = 46.00$.

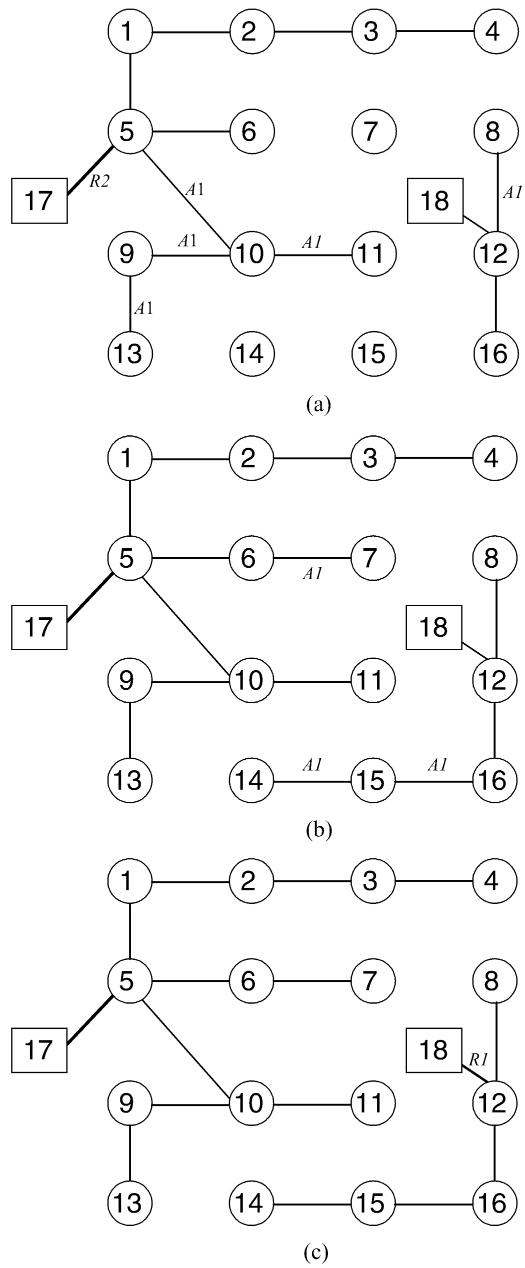

Fig. 6. Solution for multistage expansion with distributed generation considering three load levels: $C = 940.76$. (a) Stage 1: $c_1^{\mathrm{inv}} = 532.00$ and $c_1^{\mathrm{oper}} = 13.00$. (b) Stage 2: $c_2^{\mathrm{inv}} = 316.00$ and $c_2^{\mathrm{oper}} = 22.46$. (c) Stage 3: $c_3^{\mathrm{inv}} = 19.00$ and $c_3^{\mathrm{oper}} = 45.37$.

on network topology and contribute still further to reducing the search space. For the problem presented in this paper, they consisted of 135 additional constraints, distributed as shown in Table IV.

To show the effects of including the additional constraints, Table V gives a summary of the computational effort needed to find the solution to the problem of multistage planning given in Section III-B. In this table, FCT1, FCT2, and FCT3 represent the fencing constraints of types 1, 2, and 3, respectively, and NPC signifies new-path constraints. The computational effort is measured by the number of nodes evaluated in the branch-and-bound procedure, shown in the last column of Table V. Use of the constraints is indicated by the letters Y (yes) and their nonutilization by N (no).

When fencing constraints were added to the problem, together with the new-path constraints, the performance improved significantly, reducing the number of nodes evaluated by the branch-and-bound procedure by a factor of about 180. This improvement in the performance is directly related to the number of additional constraints imposed, with the best result obtained when all available constraints are used.

V. Conclusion

This paper has presented a computational evaluation of the multistage optimization model set out in [1], considering a three-stage

TABLE III
POWER GENERATIONS AND OPERATION COSTS FOR EACH STAGE AND LOAD LEVEL (LL)

Stage/LL	g_{10} [MVA]	g_{17} [MVA]	g_{18} [MVA]	c^{oper} [$]
1/LL1	—	8.40	3.60	1.63
1/LL2	—	5.04	2.16	7.04
1/LL3	—	1.68	0.72	4.33
2/LL1	1.20	12.00	6.00	5.00
2/LL2	1.92	11.04	3.60	12.46
2/LL3	—	3.36	1.20	5.00
3/LL1	6.00	12.00	8.40	17.50
3/LL2	6.52	10.50	4.56	22.87
3/LL3	—	5.76	1.68	5.00

TABLE IV
ADDITIONAL CONSTRAINTS FOR THE 18-NODE NETWORK

Constraint	Stage 1	Stage 2	Stage 3	Total
Fence Type 1	10	16	16	42
Fence Type 2	15	18	18	51
Fence Type 3	10	10	10	30
New path	12	—	—	12
Total	47	44	44	135

TABLE V
EVALUATION OF THE INFLUENCE OF ADDITIONAL CONSTRAINTS

FCT1	FCT2	FCT3	NPC	B&B nodes
N	N	N	N	5121337
Y	N	N	N	125484
Y	Y	N	N	65027
Y	Y	Y	N	55158
Y	Y	Y	Y	28179
Y	Y	N	Y	38955
Y	N	N	Y	58012

expansion planning for a distribution network having 18 connections and consisting of two substations and one node with DG capacity. Results are given for planning stage by stage, and for multistage planning with and without DG.

Comparison of the results obtained under sequential planning (one stage after the other) with results obtained under multistage planning fully justifies investment in more elaborate models which take long-term planning horizons into account. In the example discussed, there was a significant 22% reduction in cost, compared with the results obtained in Sections III-A and B.

In the mathematical model proposed here, inclusion of distributed generating capacity was simply achieved, making it possible to improve on results obtained with multistage planning. In the example used, there was a cost reduction of 10%, as shown in the results obtained in Sections III-B and C.

It was demonstrated that logical constraints, in the form of new-path and fencing constraints, significantly reduce the complexity of the combinatorial problem, such that the problem with 372 binary variables (2^{372} combinations) was solved with the evaluation of only 28179 nodes in the branch-and-bound procedure (see Table V), using the standard configurations of NEOS [6].

When the problem of planning network expansion is approached through the use of models for mathematical optimization, constraints on investment are easily incorporated and this also contributes to a reduction in the search space. Results of this paper show that the mathematical programming approach is a promising alternative where practical constraints limit investment and operation of a distribution network.

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