Off-Line Identification of PWM Driven Induction Motors Using Reference Voltages

Alcindo do Prado Junior *, Joselito Anastácio Heerdt **, Seleme Isaac Seleme Junior*

Abstract— **In this work we present a method for off-line identification of induction motors with PWM drives, i.e., the standstill estimation of all its electrical parameters. The method is achieved when we digitize the PWM driven induction motor model by considering the PWM signal from the inverter as a sequence of continuous impulses. The PWM inverter nonlinearity caused by the turn on/off time dependency of the current level in the switching transistors is modeled, allowing the method to be based only on reference voltages and measured currents. Simulation and experimental results illustrate the methodology.**

Keywords— **induction motors, discrete models, parameter estimation, system identification.**

I. INTRODUCTION

The arrival of new technologies in power electronics and digital signal processors (DSPs) has rendered possible the implementation of more and more advanced induction motors control strategies [1], [2]. These new techniques seek high dynamical performance, and, invariably, its implementation presupposes the knowledge of the induction motor electrical parameters. This fact has led to equiping the inverter with the capacity of self-commissioning, where the identification of the motor is an essential task.

The motor parameter identification can be either made off-line or on-line. The on-line identification presupposes that the motor is turning, which implies quite complex nonlinear models and the need of a very high accuracy speed measurement [3]. On the other hand, the off-line methods can be employed with the motor at standstill, thus avoiding the uncertainties due to the speed measurement and allowing the obtainment of quite simple models which permits the application of standard methods for system identification.

During the self-commissioning phase it is desirable to estimate most of the parameters from off-line experiments, leaving the on-line estimation only for the parameters which vary considerably during the inverter and motor operation, such as the rotor resistance, which varies with temperature, or the main machine inductance, wich depends upon magnetic saturation[4]. In order to increase the speed of convergence of the estimation, the initial values of the parameters for the on-line identification can be established from the off-line identification.

The self-commissioning of the inverter must presuppose that the motor identification presents certain features:

- It has to be done with the motor at standstill in order to avoid unpredictable transients caused by the controller out of tuning;
- It has to be done without generation of torque, giving that motor locking is not desirable;
- It has to be done using the inverter as the exciting element of the motor, *i.e.,* the signal that feeds the motor has to be given by Pulse Width Modulation (PWM);
- It has to present the minimum additional hardware in order to minimize costs.

This work is a continuation of [5], where we presented two discrete models for the induction motor with PWM drive, at standstill, aiming at the motor identification. The effectiveness of the models were verified only by simulation. In this work we present a method for off-line identification of induction motors based on the first model of [5], achieved when we digitize the PWM driven induction motor model by considering the PWM signal from the inverter as a sequence of continuous impulses. The PWM inverter nonlinearity caused by the turn on/off time dependency of the current level in the switching transistors is modeled, allowing the method to be based only on reference voltages and measured currents. Simulation and experimental results are shown which illustrate the employed methodology.

II. THE DISCRETE INDUCTION MOTOR MODEL

Equation (1) represents the standstill induction motor model, in terms of transfer functions, at the stationary frame [1],

$$G(s) = \frac{I_\alpha^s(s)}{V_\alpha^s(s)} = \frac{I_\beta^s(s)}{V_\beta^s(s)} = \frac{\frac{1}{\sigma L_s}s + \frac{\beta}{L_s}}{s^2 + (\alpha+\beta)s + \sigma\alpha\beta} \qquad (1)$$

where:

I_α^s = Stator current in the direct axis α;
I_β^s = Stator current in the orthogonal axis;
V_α^s = Stator voltage in the direct axis α;
V_β^s = Stator voltage in the orthogonal axis β;
R_r = Rotor coil resistance;
R_s = Stator coil resistance;
L_r = Rotor cyclic inductance;
L_s = Stator cyclic inductance;
L_{rs} = Mutual cyclic inductance;

$$\sigma = \frac{L_r L_s - L_{rs}^2}{L_r L_s}; \alpha = \frac{R_s}{\sigma L_s}; \beta = \frac{R_r}{\sigma L_r} \qquad (2)$$

Note that the decoupling among variables in the axes α and β results in two identical and independent second order systems.

* Departamento de Engenharia Elétrica, Universidade do Estado de SantaCatarina, 89.223-100 Joinville, SC, BRAZIL. E-mails: *dee2apj@joinville.udesc.br, dee2sisj@joinville.udesc.br.*

** Instituto Superior de Tecnologia, Sociedade Educacional de Santa Catarina, 89.201-972, Joinville, SC, BRAZIL. E-mail: *joselito@sociesc.com.br.*

0-7803-7474-6/02/$17.00 ©2002 IEEE

In order to estimate the Continuous Transfer Function parameters given in (1) by means of traditional identification methods, such as least squares, we need to calculate the derivatives of the measured variables, inclusively the second derivative of the stator current. This fact, highly inconvenient, has caused the development of several techniques to overcome the inherent difficulties, as discrete time approximations, all of them using sample periods which are much smaller than those required by exact discrete time models [5]. Strange enough, the exact digitization of the Transfer Function (1) had never been mentioned in literature, perhaps due to the apparent complexity of the involved algebraic manipulations, which, supposedly, could lead to equation systems with both continuous and discrete time parameters of difficult solution [6]. In [5] we developed two different methods for exact digitization of *the Transfer Function (1) and showed that the calculation* of the continuous parameters from the discrete estimates is relatively simple. In both cases it was assumed that the motor was being driven by a PWM inverter, whose switching frequency coincides with the sampling frequency of all the stator currents and voltages. The synchronization of these frequencies is something natural in digitally controlled systems and it has been even recommended [7].

Next, we present a discrete model for the induction motor, that is basically the first model of [5] with light modifications. Figure 1 presents, schematically, the motor at null speed being fed by PWM signal. In such condition a voltage u_k to be applied should be small because the current is limited only by the stator resistance. This fact implies, necessarily, that the *duty-cycle* of the *PWM* signal be very small. Thus, the *PWM* signal can be viewed as a sequence of impulse functions of amplitude u_k (the pulse area) and delays $\lambda_k = \frac{|u_k|}{2V_{DC}}T$, where T is the sampling period (equal to the inverter switching period) and V_{DC} is the DC link voltage of the inverter.

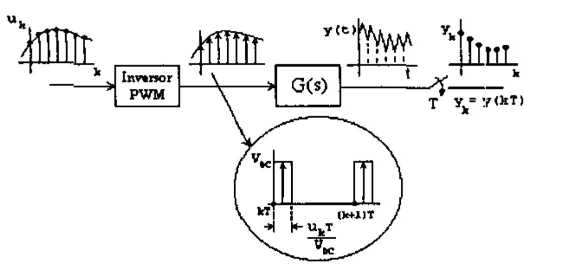

Fig. 1. PWM modulator functioning as an impulse function generator

In [5] the delay λ_k was considered in order to obtain the discrete model. Experimental results have been showing that the model becomes more robust, for identification purposes, if that delay is not considered. So, it is quite easy to show that the Discrete Transfer Function for this system will be obtained from the Z Transform of $G(s)$ [8]:

$$G(z) = T.Z[G(s)] \tag{3}$$

The expressions (1) and (3), following some algebraic manipulation, can be written as

$$G(z) = \frac{b_1 z + b_2}{z^2 + f_1 z + f_2} \tag{4}$$

where

$$f_1 = -e^{-\frac{T}{2}(\alpha+\beta)}(e^{-\frac{T}{2}t_2} + e^{\frac{T}{2}t_2}) \tag{5}$$

$$f_2 = e^{-(\alpha+\beta)T} \tag{6}$$

$$t_2 = \sqrt{(\alpha+\beta)^2 - 4\sigma\alpha\beta} \tag{7}$$

The coefficients b_1 and b_2 are somewhat arduous to be obtained. For that sake we define the auxiliary variable t_1 as

$$t_1 = -\frac{\log(f_2)}{T} = \alpha + \beta \tag{8}$$

From Eqs. (5), (6), (7) and (8), t_2 can be written as a function of coefficients f_1 and f_2, as:

$$t_2 = -\frac{2\log\left(\frac{-\frac{f_1}{\sqrt{f_2}} + \sqrt{\frac{f_1^2}{f_2} - 4}}{2}\right)}{T} \tag{9}$$

Coefficients b_1 and b_2 are given then as

$$\begin{aligned} b_1 = \frac{T}{2\sigma L_s t_2}[(t_1 + t_2 - \tfrac{t_1^2 - t_2^2}{2\alpha})e^{-\frac{T}{2}(t_1+t_2)} \\ +(\tfrac{t_1^2 - t_2^2}{2\alpha} - t_1 + t_2)e^{-\frac{T}{2}(t_1-t_2)}] \end{aligned} \tag{10}$$

$$b_2 = -\frac{T}{\sigma L_s}e^{-Tt_1} \tag{11}$$

A. Remark

• This digitization method foresees that the ripple due to the PWM signal is part of the stator current at the output of the system. This is an important assumption, once that the ripple is no longer considered as a perturbation for the discretized system, but as a part of the signals.

III. OBTAINMENT OF THE MOTOR PARAMETERS

The Discrete Time Transfer Function (4) can be identified using classical methods of parameter estimation, such as the least squares, the extended least squares, the maximum likelihood, the output prediction error, among other methods to obtain estimates for the four parameters b_1, b_2, f_1 and f_2. Then, it is quite easy to estimate from them, the four parameters of the Continuous Time Transfer Function (1): α, β, σ and L_s.

First, notice that the auxiliary variables t_1 and t_2 can be easily calculated from f_1 and f_2. Notice also that the mathematical relation between b_1 and b_2 depends only on α, which can be then calculated, resulting in

$$\alpha = \frac{\frac{t_1^2 - t_2^2}{2}[t_4 - t_3]}{(t_1 - t_2)(\frac{b_1}{b_2}t_3t_4 + t_4) - (t_1 + t_2)(\frac{b_1}{b_2}t_4t_3 + t_3)} \tag{12}$$

where

$$t_3 = e^{-\frac{T}{2}(t_1+t_2)} \tag{13}$$

$$t_4 = e^{-\frac{T}{2}(t_1 - t_2)} \qquad (14)$$

The values of β and σ can be calculated from (7) and (8), as

$$\beta = t_1 - \alpha \qquad (15)$$

$$\sigma = \frac{t_1^2 - t_2^2}{4\alpha\beta} \qquad (16)$$

The expression (11) gives the value of the L_s estimate,

$$\hat{L}_s = -\frac{T}{\sigma b_2} e^{-Tt_1} \qquad (17)$$

Now, we will seek the estimates for the other parameters of the induction motor, that is to say, of R_s, R_r, L_r and L_{sr}, from the three equations in (2). The non identificability problem for the induction motor appears, once the number of equations is smaller than the number of parameters. In order to overcome this problem, inherent to the induction motor when we just have access to the stator signals, we introduced the concept of motors classes, defined by the norm [9]. This norm classifies the induction motors according to its torque-speed characteristics, in function of the departure current, adding the following equation

$$L_s - L_{sr} = k(L_r - L_{sr}) \qquad (18)$$

where:
$k = 1$, for motors of classes A, D and with wound rotor;
$k = \frac{2}{3}$, for motors of class B;
$k = \frac{3}{7}$, for motors of class C.

The equations (18) and (2) result in the L_r estimate,

$$\hat{L}_r = \frac{2k + (1-k)^2(1-\sigma) - \sqrt{[2k + (1-k)^2(1-\sigma)]^2 - 4k^2}}{2k^2} \hat{L}_s \qquad (19)$$

For $k = 1$ (the most common case), the equation (19) becomes

$$\hat{L}_r = \hat{L}_s \qquad (20)$$

The other equations in (2) allow then to determine the R_s, R_r and L_{sr} estimates,

$$\hat{R}_s = \hat{\alpha}\hat{\sigma}\hat{L}_s; \qquad \hat{R}_r = \hat{\beta}\hat{\sigma}\hat{L}_r \qquad (21)$$

$$\hat{L}_{sr} = \sqrt{(1-\hat{\sigma})\hat{L}_s\hat{L}_r} \qquad (22)$$

IV. CORRECTION OF THE REFERENCE VOLTAGE

Once the duty-cycle of the PWM signal is small, the pulse widths are necessarily influenced by the delay effects caused by the switching of the electronic devices of the inverter [10], causing voltages V_α^s and V_β^s different from the PWM reference voltage V_{ref}^s. In order to avoid the inclusion of voltage sensors to measure V_α^s and V_β^s , the stator voltage should be compensated to automatically correct the distortion, because such sensors are expensive and pose some difficulty for a good acquisition of the pulsed signals of the PWM.

As proposed by [11], the inverter distortion model will be obtained from the static behavior between the reference voltage and the stator currents. For static signals , the transfer function (1) becomes $G(s) = 1/R_s$. Then, Figure 2 presents the inverter distortion nonlinear model based on static measurements of the stator currents I_α^s or I_β^s. In this Figure the static voltages V_α^s or V_β^s are denoted by v^s and the static currents I_α^s or I_β^s are denoted by i^s.

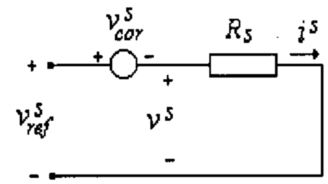

Fig. 2. Static Model for the PWM Inverter and Motor

Thus, we have

$$v_{ref}^s = R_s i^s + v_{cor}^s(i^s) \qquad (23)$$

Figure 3 shows a typical v_{ref}^s x i^s curve. For negative currents this curve is symmetrical negative.

For several measured values of i^s, a least square polynomial fit of the equation (23) gives the coefficients of the polynomial

$$v_{ref}^s = a_0 + a_1 i^s + a_2(i^s)^2 + \ldots + a_n(i^s)^n \qquad (24)$$

The value of R_s can be obtained from the curve rate at a current i_o^s sufficiently high,

$$R_s = \left[\frac{dv_{ref}^s}{di^s}\right]_{i_o^s} = \left[a_1 + 2a_2 i^s + \ldots + na_n(i^s)^{n-1}\right]_{i_o^s} \qquad (25)$$

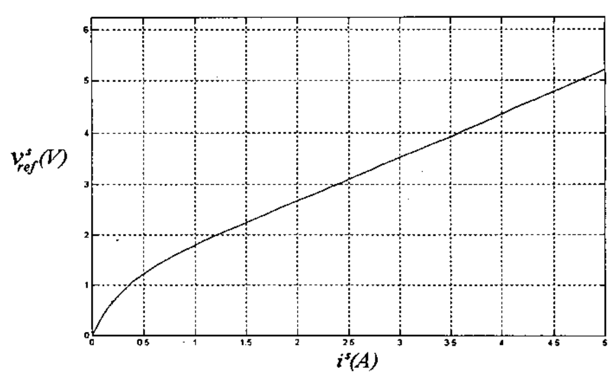

Fig. 3. Typical v_{ref}^s x i^s Curve

So, the correction curve v_{cor}^s will be given as a function of the measured stator current by

$$v_{cor}^s(i^s) = a_0 + a_1 i^s + a_2(i^s)^2 + \dots + a_n(i^s)^n - R_s i_s \quad (26)$$

V. SIMULATION RESULTS

In this section some simulation results are shown for a motor with the following nominal characteristics: *power = 3.0cv, speed = 1710 rpm, voltage=220 V, current = 8.09 A;* its model has the following parameters: $R_s = 0.84\ \Omega$, $R_r = 0.49\ \Omega$, $L_s = L_r = 0.065\ H$, $L_{rs} = 0.062\ H$.

Figure 4 shows the Simulink identification setup. The PWM inverter, supposed without distortion ($v_{cor}^s = 0$) was excited by a Pseudo Random Binary Signal (PRBS) with *amplitude* $= 5\ V$ and *period* $= 0.1s$. In order to render the simulation more realistic, a gaussian white noise has been added to the stator current measurements.

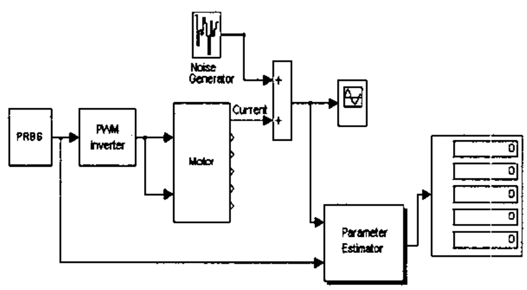

Fig. 4. Simulink identification setup used in the simulation

The parameter estimation of the discrete model was accomplished with the Output Error method from MATLAB Identification Toolbox (*oe* command). 2048 points were used, at *sampling time* $= 0.001s$, corresponding to a data acquisition time interval of 2.1 s.

The current response is presented in Figure 5. Notice that the signal is quite noisy, due mainly to the ripple caused by the PWM.

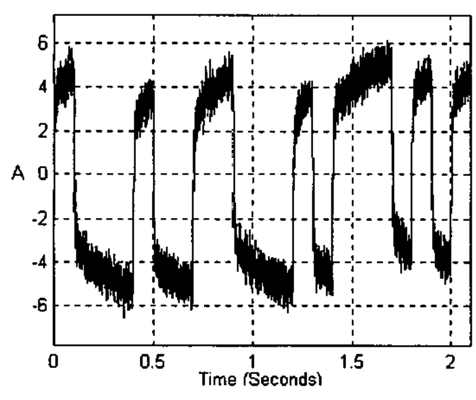

Fig. 5. Simulation current corresponding to the PRBS input

The application of the proposed method resulted in the following estimates for the electrical parameters of the motor:

$\hat{R}_s = 0.838\ \Omega$, $\hat{R}_r = 0.488\ \Omega$,
$\hat{L}_s = \hat{L}_r = 0.0648\ H$, $\hat{L}_{rs} = 0.0617\ H$

The accuracy of these estimates is better than 0.5%, even with the high added noise, which shows the method is insensible to the existence of the ripple. In [12] a lot of simulations were realized, for several sampling rates and several noise levels, confirming the applicability of the method.

VI. EXPERIMENTAL RESULTS

In this section some experimental results are shown for the same motor simulated in the above section. The platform used for accomplish the experimentations is depicted in Figure 6.

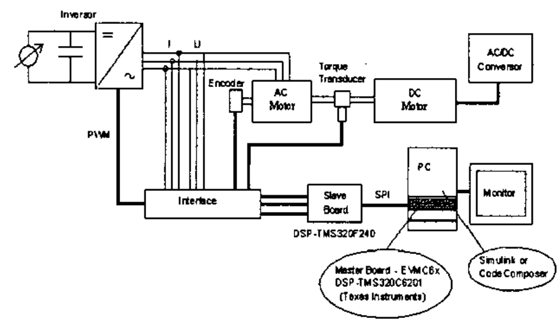

Fig. 6. The platform used for experimentation

This platform uses two DSPs (TMS320F240 and TMS320C6201) from Texas Instruments, in a master-slave configuration. The first one is just dedicated to input-output tasks, while the second one is dedicated to control issues. The communication among them is made through the SPI protocol (Serial Peripheral Interface) at $1.1Mbps$. The programming of the experiments can be realized using Simulink or C++. Further more information about this platform can be found in [13] and [14].

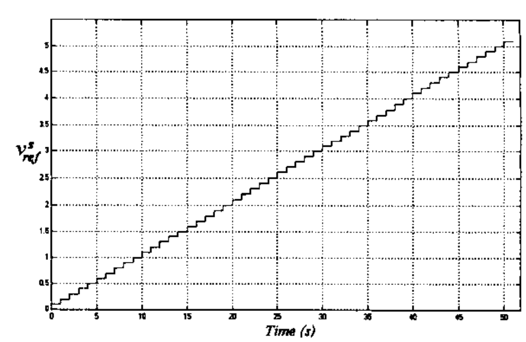

Fig. 7. V_{ref}^s used for Correction Estimation

A. Determination of the inverter correction curve

Figure 7 shows the reference voltage V^s_{ref} that we should use for estimate the inverter correction curve. This signal is composed by 50 steps with $amplitude = 0.1V$ and $duration = 1,0s$.

The motor current response is depicted by Figure 8.

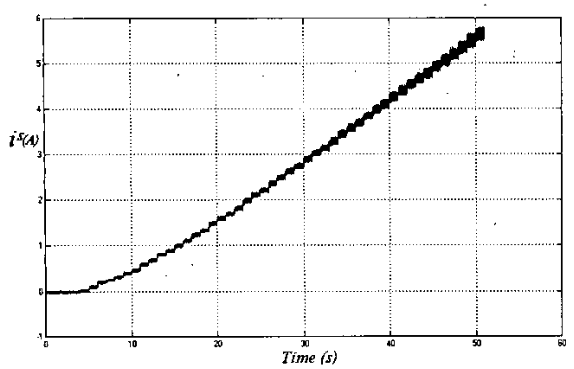

Fig. 8. Current response i^s corresponding to V^s_{ref}

The *duration* of $1,0s$ is enough so that the answer to each step is stabilized. Using V^s_{ref} and i^s, sampled at $1,0s$, a 7^{th}order least square polynomial fit of the equation (24) gives the coefficients:

$a_7 = 0.00170919$; $a_6 = -0.0344647$; $a5 = 0.2791116$; $a_4 = -1.163280$; $a_3 = 2.663295$; $a_2 = 3.34100$; $a_1 = 3.13065$ and $a_0 = 0.31513128$.

The equation (25) gives the mean value

$$\hat{R}_s = 0.838\Omega$$

for ten values of i^s taken between $3.0A$ and $5.0A$.

Figure 9 shows the curve v^s_{ref} x i^s generated by the equation (24), whose inclination for $3 < i^s < 5$ is worth $\hat{R}_s$.

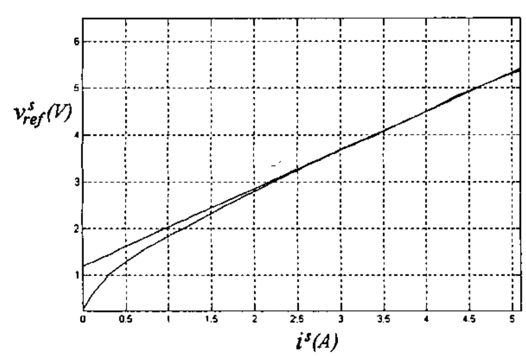

Fig. 9. Experimental v^s_{ref} x i^s Curve

The correction curve (26) for the inverter distortion then can be determined, whose graph is shown by Figure 10.

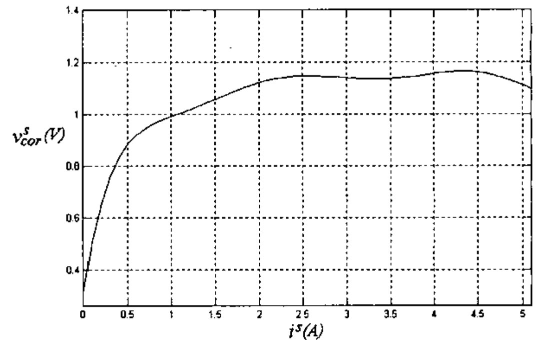

Fig. 10. Correction Curve for the Inverter Distortion

B. Motor parameters estimation

Figure 11 shows the Simulink programming used to apply the PRBS signal (with *amplitude = 5.8 V* and $period = 0.1s$) to the motor and to acquire the response of the tree-phase currents, that are stored in files.

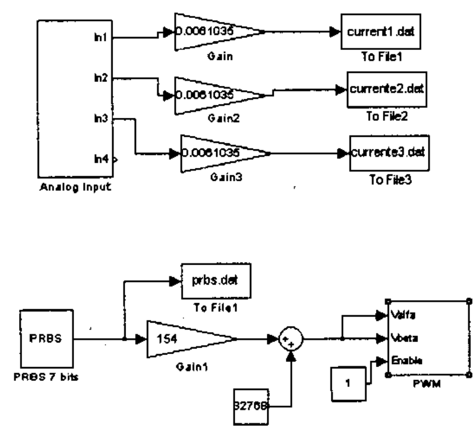

Fig. 11. Simulink experimentation setup.

The parameter estimation is made off-line by the Simulink programming presented in the Figure 12.

In this setup, the tree-phase currents first are transformed to two-phase currents, in the stationary frame, by the K_{ab} block. The Fcn Block accomplishes the correction given by the equation (26).

The Figure 13 shows a typical experimental two-phase stator current for the used PRBS signal.

Ten experiments resulted in the following estimates:

	Mean	Standard Deviation
R_s	0.839	0.019
R_r	0.496	0.013
L_s	0.0670	0.043
L_r	0.0670	0.0043
L_{sr}	0.0638	0.0042

Similar to the simulation, the experimental results confirm the potential of the method.

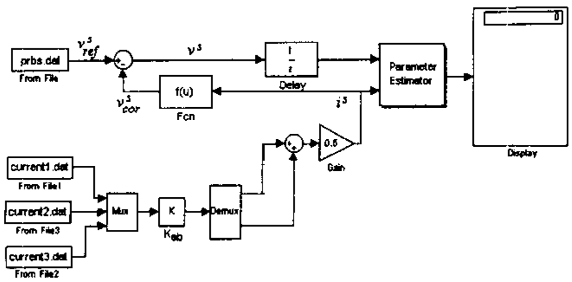

Fig. 12. Simulink identification setup

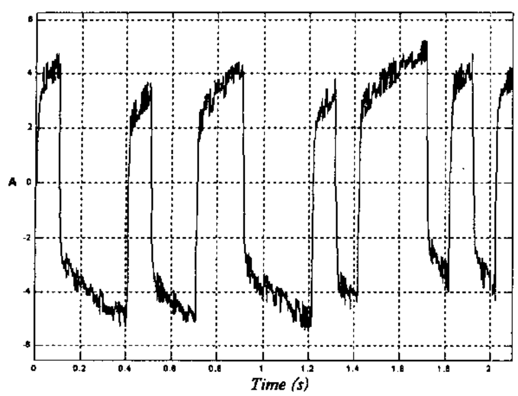

Fig. 13. Experimental two-phase current corresponding to the PRBS input

VII. CONCLUSION

In this work we have presented a method for off-line identification of induction motors with PWM drives, *i.e.*, the standstill estimation of all its electrical parameters. The method was achieved when we digitize the PWM driven induction motor model by considering the PWM signal from the inverter as a sequence of continuous impulses. The PWM inverter nonlinearity caused by the turn on/off time dependency of the current level in the switching transistors was modeled, allowing the method to be based only on reference voltages and measured currents. Simulation examples and experimental results confirm the applicability of the method.

REFERENCES

[1] W. Leonhard, *Control of Electric Drives.* Springer-Verlag, 1996.

[2] B. K. Bose, *Power Electronics and Variable Frequency Drives.* IEEE Press, 1997.

[3] H. Borsting, M. Knudsen, and P. Vadstrup, "Standstill estimation of electrical parameters in induction motors using an optimall input signal," in *Proc. of the Conference on European Power Electronics and Applications*, vol. 1, (Sevilla), pp. 814–819, 1995.

[4] R. Krishnan and F. C. Doran, "Study of parameter sensitivity in high performance inverter-fed induction motor drive systems," *IEEE Trans. on Industry Application*, vol. 23, no. 4, pp. 623–635, 1987.

[5] A. PradoJr, S. I. SelemeJr, and J. A. Heerdt, "Discrete modelling for off-line identification of PWM driven induction motors," (Guimarães, Portugal), pp. 307–312, Portuguese Association of Automatic Control, 4th Portuguese Conference on Automatic Control, october 2000.

[6] P. J. R. de Oliveira, P. F. Seixas, and L. A. Aguirre, "Aplicação do modelo contínuo no tempo à estimação de parâmetros de máquina de indução," in *Congresso Brasileiro de Automática*, vol. 3, (Uberlândia), pp. 807–812, september 1998.

[7] V. Blasko, V. Kaura, and W. Niewiadomski, "Sampling of discontinuous voltage and currents in electrical drives: A system approach," *IEEE Trans. on Industry Application*, vol. 34, pp. 1123–1129, september/october 1998.

[8] K. Aström and B. Wittemark, *Computer Controlled Systems.* Prentice-Hall, Inc., 1984.

[9] IEEE-112, "Standard test procedure for polyphase induction motors generators," 1996.

[10] A. Bünte and H. Grotstollen, "Parameter identification of an inverter-fed induction motor at standstill with a correlation method," in *Proc. of the Conference on European Power Electronics and Applications*, vol. 5, (Brighton), pp. 97–102, 1993.

[11] H. Rasmussen, M. Tonnes, and M. Knudsen, "Inverter and motor model adaptation at standstill using reference voltages and measured currents," in *Proc. of the Conference on European Power Electronics and Applications*, vol. 1, (Sevilla), pp. 367–372, 1995.

[12] J. A. Heerdt, "Identificação de motores de indução para autocomissionamento de controles avançados," Master's thesis, Universidade do Estado de Santa Catarina - UDESC, december 2000.

[13] F. E. Rosa, "Uma proposta de plataforma para ensaios de controle de sistemas usando DSPs," Master's thesis, Universidade do Estado de Santa Catarina - UDESC, september 2000.

[14] A. H. Sousa, A. J. Zipf, A. PradoJr, and A. Nied, "Plataforma para acionamento e controle de motores de indução usando DSPs," (Natal, Brazil), submitted to the Congresso Brasileiro de Automática, september 2002.